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Numerical investigation on hydrodynamic developement length of gaseous flow in parallel plate micro channel

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Abstract : Fluid flows in the entrance region of micro channels have been investigated using fourth order finite difference implicit scheme Considerations have been given to investigate the compressibility and rarefaction effects assuming slip and no slip on developing laminar flow at very close to the entry It has been seen that the effect of compressibility is important for higher Reynolds number (*Re*) and rarefaction is significant for lower reynolds number *Re*. The result suggests that rarefaction has a significant effect on hydrodynamic development length A new relation has been proposed for entry length for Reynolds number in between 1 to 400 The proposed relation provides an efficient, practical and accurate toot in predicting the hydrodynamic development length It has also been observed that due to slip flow mass flow rate increases

Keywords : Hydrodynamic developement length, Reynolds number, Knudsen number

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1. Introduction

Small channels (D_h < 1 mm) have been used in a large variety of novel applications, such as heat exchangers, micro sensors, biological cell reactors and selective membranes. The study of fluid transport phenomena in small channels on the scale of micro-electro-mechanical systems (MEMS) have been an interesting research topic not for a very long time. At these length scales the dimensions of the system itself become comparable to the mean free path of the particles flowing. For liquid, this in turn tends to make the flow granular in nature, whereas for gases the flow becomes

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rarefied. Studies reflect that the continuum approach with no-slip wall conditions can no longer predict the fluid flow characteristics in micro devices. As the flow becomes molecular, a number of parameters such as : surface force, roughness, rarefaction, viscous force, thermal creep start playing significant role which are not dominant in macroscopic flows.

The degree of rarefaction and applicability of continuum model is determined by local Knudsen number and is defined as, the ratio of molecular mean free path λ to the characteristic length L. This means that rarefaction becomes an important phenomenon and accordingly, micro scale flow is no longer in the continuum flow regime. Therefore, conventional continuum computational fluid dynamics (CFD) method with no-slip wall boundary conditions are not suitable. First order slip boundary conditions should be applied in the Navier-Stokes equations in the range of $0.001 \leq K_n \leq 0.1$. The transition region spans the range of $0.1 \leq K_n \leq 10$ and second order slip boundary conditions are applicable.

Nomenclature			
H	Channel height		
u,v	Velocity components in axial and transverse directions	n	Number of time steps
u_0	Free stream velocity		
Ma	Mach number		
R	Gas constant	α	Aspect ratio (W/H)
D_h	Hydraulic diameter	ρ	Density
L_e	Hydrodynamic development length		
P_r	Pressure ratio = Inlet pressure/Outlet pressure	μ	Viscosity
T_{gas}	Temperature of gas adjacent to wall	λ	Molecular mean free path
T_{wall}	Wall temperature	τ_w	Wall shear stress
T	Temperature	γ	Ratio of specific heats
Re	Reynolds number	σ_v	Tangential momentum accommodation coefficient
ma	Mach number		
Kn	Knudsen number		

Over the past two decades researches are going on to determine the fluid flow characteristics in micro devices. Pfahler *et al* [1], Harley *et al* [2] reported that conventional analyses are unable to predict the transport of gas in silicon micro-machined channels. Harley *et al* [2], focused on nonlinear effects of gaseous slip flows. Peng *et al* [3] found experimentally that transition to turbulent flow begins at Re = 200 to 700 and that fully turbulent flow occurs when Re is in between 400 to 1500. Chen *et al* [4] studied the flow of nitrogen and helium in micro channel. They concluded that due to larger wall shear stress, pressure gradient is high even at lower velocities. The capability of Navier-Stokes equations to adequately represent the flow and heat transfer behavior in the micro channels has been called into question in some of these studies.

The focus of the analytical and computational models cited above were mainly limited to flow characteristics in the hydro dynamically developed zone with the exception of the work of Atkinson *et al* [5] and Chen [6] who however assumed no-slip flow (*i.e* $Kn = 0.0$) There are a number of applications where the entrance region fluid flow characteristics become important There is a clear need for additional investigations Most notably, flow through micro geometry has become a topic area of the utmost research activity and increased interest with the dawn of the new century This has motivated the present work where a study of the fluid flow characteristics in the entrance region of micro channels has been carried out The present study examines the role of Reynolds number and Knudsen number on the hydrodynamic development length at the entrance of the parallel plate micro channel

2. Governing equations

The channel is assumed to be long enough such that fully developed flow may safely occur The channel is wide with an aspect ratio ($\alpha = W/H$) of approximately 40 Since the three dimensional effects are proportional to the inverse of the α^2 [7], The flow can be safely assumed to be two dimensional

The governing equations for this study are

The continuity equation .

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0 \tag{1}$$

The unsteady two-dimensional compressible Navier-Stokes equations

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{4}{3}\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{1}{3}\frac{\partial^2 v}{\partial x \partial y}\right), \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{4}{3}\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}+\frac{1}{3}\frac{\partial^2 u}{\partial x \partial y}\right) \tag{3}$$

The equation of state .

$$P = \rho RT \tag{4}$$

The above equations are non dimensional zed by introducing

$$x^* = \frac{x}{H}, y^* = \frac{y}{H}, u^* = \frac{u}{u_0}, v^* = \frac{v}{u_0}, \rho^* = \frac{\rho}{\rho_0}, P^* = \frac{P}{\rho u_0^2}, T^* = \frac{T}{T_0} \tag{5}$$

3. Boundary conditions

At the inlet a uniform velocity profile has been assumed Therefore $\partial u/\partial x = 0$ and $v = 0$. The pressure at the inlet is specified by pressure ratio. The temperature at the inlet has been assumed to be same as the wall temperature. At the outlet, the axial gradients of the flow variables has been set to zero.

4. Model descriptions

The cross section of the micro channel is shown in Figure 1 Several experimental and numerical results have been documented for this popular geometry Table 1 lists the dimensions of the micro channel and relevant physical properties of the working fluid

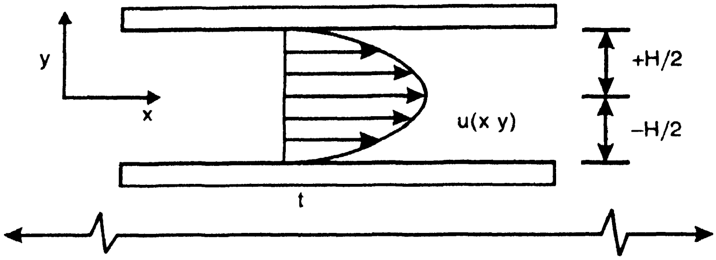

Figure 1 Geometry of micro channel analysis

Table 1 Model dimensions

Flow parmeters	Range or mean value
Length L	3000 μm
Width W	40 μm
Height H	1 2 μm
Pressure ratio $P_r = P_{in}/P_{out}$	1 35 1 50 1 95 2 20 2 35, 2 51 2 7
Gas temperature at inlet T_i	300 K
Wall temperature T_w	300 K
Outlet mach number	$(2\ 25\text{–}25) \times 10^{-4}$
Outlet Kundsen number	$0\ 001 \le Kn \le 0\ 1$
Reynolds number	1–400

5. Hydrodynamic entrance length

When a viscous fluid enters in a duct, a velocity boundary layer develops along the inside surfaces of the duct The boundary layer fills the entire duct gradually The region where the velocity profile is developing is called the hydrodynamic entrance region and its extent is the hydrodynamic entrance length For practical engineering calculations, the hydrodynamic development length (L_e) is arbitrarily defined as the axial distance required for centre line velocity to reach 99% of the fully developed value The region beyond the entrance region is referred to as hydro dynamically fully developed region In this region, the boundary layer completely fills the duct and velocity profile becomes invariant with the axial co-ordinate

The important features of gas flow in micro ducts are mainly due to rarefaction and compressibility effects The inertia force is negligible compared with the viscous force for small Reynolds number If $Ma\ Kn << 1$, all stream wise derivatives can be ignored except the pressure gradient, which is the only driving force. Hence, the fully developed assumptions imply that the velocity profile can be determined by solving ·

$$\mu \frac{d^2u}{dy^2} = \frac{dP}{dx} \tag{6}$$

Integrating eq. (6) twice with respect to y, assuming $P = P(x)$, Arkilic *et al* [9] the derived following fully developed velocity profile :

$$u(y) = \frac{H^2}{8\mu}\frac{dP}{dx}\left[1-\left(\frac{2y}{H}\right)^2 + 4Kn(x)\right]. \tag{7}$$

In addition the maximum velocity along the centerline of the duct ($y = H/2$) can be derived as

$$u_{max} = \frac{1.5\bar{u}\left(1 = 8\frac{2-\sigma}{\sigma}Kn\right)}{1+12\frac{2-\sigma}{\sigma}Kn}. \tag{8}$$

As $Kn \rightarrow 0.0$ the above equation reverts to familiar Poiseuille flow. $u_{max} = 1.5\bar{u}$.

6. Slip flow

Slip flow occurs when gases are at low pressure or for flow in micro passages. In presence of wall-normal and tangential temperature gradients, for an ideal gas the complete slip-flow (first order) will be in no dimensional form [2] :

$$u^*_{gas} - u^*_{wall} = \frac{2-\sigma_v}{\sigma_v}Kn\left(\frac{\partial u^*}{\partial y^*}\right)_w + \frac{3}{2\pi}\frac{\gamma-1}{\gamma}\frac{Kn^2Re}{Ec}\left(\frac{\partial T^*}{\partial x^*}\right)_w. \tag{9}$$

The second term of the above equation is known as *'thermal creep'* which generates slip velocity in the direction opposite to the increasing temp. For isothermal wall the above equation becomes :

$$u^*_{gas} - u^*_{wall} = \frac{2-\sigma_v}{\sigma_v}Kn\left(\frac{\partial u^*}{\partial y^*}\right)_w. \tag{10}$$

For continuum flow, $Kn \rightarrow 0$, above equation reduces to the familiar no-slip conditions. $\sigma_v = \frac{\tau_i - \tau_r}{\tau_i - \tau_w}$ Tangential momentum accommodation coefficient (TMAC). Subscripts i, r, w stands for incident, reflected and solid wall conditions respectively; τ is a tangential momentum flux. For real walls, a portion of the momentum of the incident molecules is lost to the wall and a portion is retained by the reflected molecules. TMAC is defined as the fraction of molecules reflected diffusively. The value of this coefficient depends on the fluid, the solid, temperature, local pressure and surface finish and has been determined experimentally to be in between 0.2 and 0.8. The lower limit is for exceptionally smooth surface while the upper limit is typically of most practical surface.

Recently Arkilic *et al* [9] experimentally concluded that TMAC varies in between 0.75 to 0.85.

7. Methodology

The governing equations are nonlinear in nature. The numerical computations were as carried out by solving the governing equations along with boundary conditions by forth order accurate finite difference implicit scheme with uniform grids. Forward difference in time and central difference in space has been used. Convergence and accuracy were obtained by MacCormack multi step (intermediate step between n and $n+1$) scheme. Stability has been checked for every grid points for each iteration by the following empirical formulas proposed by *Tannehill et al* [10].

$$\Delta t \leq \frac{\sigma(\Delta t)_{CFL}}{1+2/\mathrm{Re}_{\Delta}} \text{ where } \sigma \cong 0.7-0.9\ , \tag{11}$$

$$(\Delta t)_{\mathrm{CFL}} \leq \left[\frac{|u|}{\Delta x}+\frac{|v|}{\Delta y}+a\sqrt{\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}}\right]^{-1}, \tag{12}$$

$$Re_{\Delta}=\min\left(Re_{\Delta x}, Re_{\Delta y}\right) \geq 0,\ Re_{\Delta x}=\frac{\rho|u|\Delta x}{\mu},\ Re_{\Delta y}=\frac{\rho|v|\Delta y}{\mu} \tag{13}$$

The flow is assumed to be two dimensional and flow variation in z-direction is assumed negligible. The governing equations for this study are the continuity equation, unsteady two-dimensional compressible Navies-Stokes equation and the equation of state. Two types of boundary conditions on the channel wall, a slip wall and a no-slip wall has been employed in this simulation to study their effects on the micro channel flow. The governing equations along with the boundary conditions are solved using a numerical code written in FORTRAN-90 language to get velocity and pressure at every internal point. The solution procedure starts by applying the initial guesses values and obtained a converged solution by iteration. The iterations were continued until the sum of residuals for all computational cells became negligible *i.e.* $\max\left|\left(u^{n+1}-u^{n}\right)/u^{n+1}\right| \leq 10^{-5}$ is satisfied, where u is the velocity in the x direction and n denotes the number of the time steps. The distribution of cells in the computational domain was determined from a series of tests with a different number of cells in x and y directions. A uniformly distributed 5000 × 30 grid is found to be adequate and has been used throughout the study.

8. Results and discussions

A thorough investigation at very close to the entry has been performed by varying the different mass flow rates for nitrogen flow through the channel under isothermal conditions. The results are first compared with the available experimental and numerical results of prominent researchers. Then they were applied to the rarefied flow in the hydrodynamic flow region has been investigated.

In channel flows the velocity of the molecules near the wall decreases and the molecules near the centerline accelerated to converse the mass flow rate Figure 2 shows the axial velocity at three different cross sections with a pressure ratio of 2 65

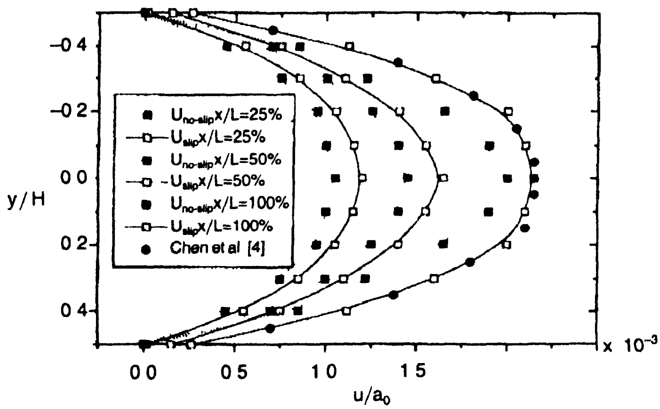

Figure 2. Velocity profile along the transverse direction of nitrogen flow at three locations subjected to slip and no slip boundary conditions, P_r = 2 65

The axial velocity has been non-dimensional zed with the velocity of sound To check the accuracy of the present work, the fully developed velocity profile has been compared with the numerical results of Chen *et al* [4] It is important to note that Chen *et al* [4] utilized an explicit finite difference method on a 6000 × 23 grids, while 5000 × 30 grids have been used for present numerical simulations Nitrogen was used as the working fluid in the micro channel The geometrical dimensions and other parameters used for nitrogen flow have been shown in Figure 1 and listed in Table 1. It is also visible from this figure that at downstream velocity increases with a corresponding increase in wall velocity due to slip effect In order to satisfy the conservation of mass along the channel direction, the flow is accelerated to make up the density drop that result from the decrease of the pressure In case of compressible flow the velocity is obviously accelerated because of local bulk density decreases due to low local pressure there and the average velocity increases. This is due to the volume expansion of the gas caused by the pressure drop. Consequently fully developed flow never occurs due to this compressibility effect even if the micro channel is very long or the length is very large. In this present model it has been assumed that if the axial velocity gradient is less that 10^{-6}, the flow is fully developed At inlet, uniform velocity distribution has been assumed.

Hydrodynamic development lengths were computed for a wide range of Knudsen numbers (0 001 < *Kn* < 0 1) and a range of Reynolds number in between 1 to 400 Some of them are presented here. Figure 3 shows the non-dimensional development length as a function of *Re*. The results of the present investigation have been super imposed on the results of Atkinson *et al* [8]. Atkinson *et al* [8] found that hydrodynamic

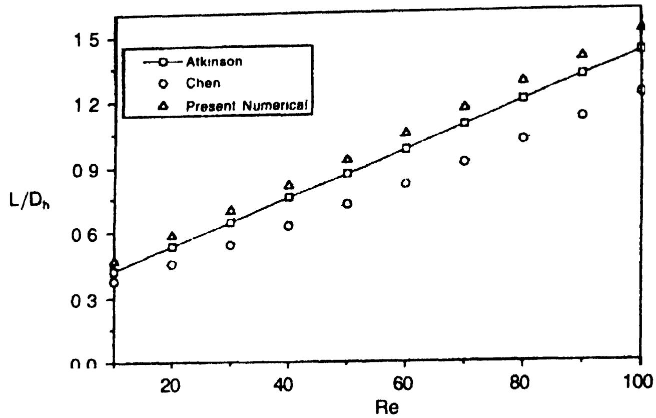

Figure 3. Comparison variation of development length $Kn = 0\ 0$

development length for parallel-plate duct flow at lower Reynolds number can be related by the following linear relations :

$$\frac{L_e}{D_H} = 0.3125 + 0.011\,Re \tag{14}$$

However their calculations were based on $Kn = 0.0$, that means there results are based on continuum flow.

However Chen [6] proposed that the above relation could be related to :

$$\frac{L_e}{D_H} = \frac{0\ 315}{0.0175\,Re + 1} + 0.011\,Re \tag{15}$$

Figure 3 indicates that the results obtained from the present numerical investigation at $Kn = 0$ are in good agreement with the results of Atkinson *et al* the deviation is only ~5%. More importantly the present results show that as the Reynolds number increases the hydrodynamic development length also increases. From Figure 4 it is

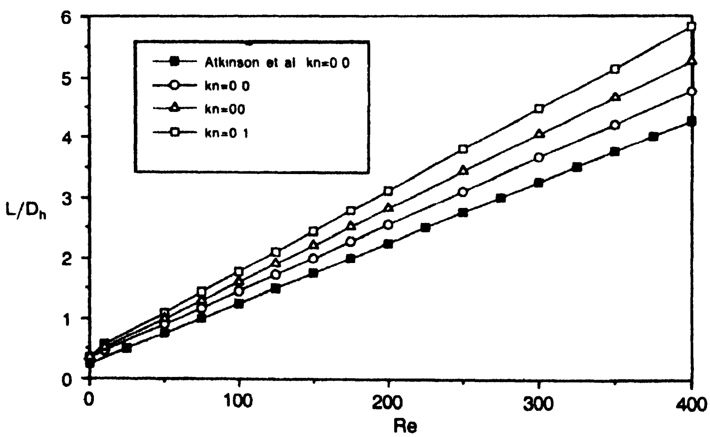

Figure 4. Variation of development length for rarefied slip flow in parallel plate microchannel

also clear that *Kn* has a significant role on development length. As *Kn* increases development length also increases. It has also been observed that, for Reynolds number 400, the entrance length when $Kn = 0.1$ is 21.7% longer that corresponding no-slip solutions (*i.e.* $Kn = 0.0$). The formulae developed by Atkinson *et al* and Chen are not valid in the slip flow regime Therefore consequently a new development length equation must be evaluated. Based on numerical calculations over a wide range of Reynolds number ($1 < Re < 400$) and Knudsen number with in the slip flow regime using curve fitting methods we propose a new relation for hydrodynamic length has been suggested as follows :

$$\frac{L_e}{D_H} = 0.350\left(\frac{1+Kn}{1-Kn}\right) + 0.011\left(\frac{1+Kn}{1-Kn}\right).Re. \tag{16}$$

It can be seen that the proposed equation provides a good representation of the numerical development length data. The proposed entrance length equation should be used in the laminar (Re = 1 to 400) flow regime only

9. Conclusions

A finite difference based micro scale formulation has been developed and compared with the available experimental and numerical results for two dimensional compressible flow. The investigation is mainly concentrated on the low Reynolds number behavior at the entrance of parallel plate micro channel. A new relation for hydrodynamic entry length has been proposed. The numerical model has indicated that the Knudsen number has a significant effect on the hydrodynamic development length. As Knudsen number increases hydrodynamic development length increases and nonlinear pressure distribution becomes more pronounced.

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